PROBLEM SET 7. DUE THURSDAY, 14 SEPTEMBER


Supplementary reading. Simmons, sections 7.2 and 10.7.

1. (3pts) Compute the following definite integrals.
   (a) \( \int_0^1 x(x^2 + 2)^3 \, dx \)
   (b) \( \int_0^1 xe^x \, dx \)
   (c) \( \int_0^2 \sqrt{4 - x} \, dx \)

2. (3pts) Find the geometric area of the following functions on the corresponding interval.
   (a) \( f(x) = 6 - 3x^2 \) on \([0, 2]\)
   (b) \( f(x) = 3x^2 - 3 \) on \([0, 3]\)
   (c) \( f(x) = 9x^2 - 36 \) on \([0, 4]\)

3. (8pts) Compute the following integrals using integration by parts.
   (a) \( \int \ln(x) \, dx \)
   (b) \( \int x^2e^x \, dx \) (You will have to do the process twice in this example.)
   (c) \( \int xe^{ax} \, dx \) for a real number \( a \)
   (d) \( \int (\ln(x))^2 \, dx \)

4. (3pts) Find the (geometric) area between the following curves and the \( x \)-axis.
   (a) \( f(x) = 27 - 3x^2 \)
   (b) \( f(x) = 12 - \frac{3}{2}x^2 \)
   (c) \( f(x) = -2x - \frac{x^2}{2} \)

5. (3pts) Find the area of the region bounded by the two curves given.
   (a) \( f(x) = \cos(x) \) and \( g(x) = \sin(2x) \) on \([0, \frac{\pi}{2}]\) (Hint: \( f(x) = g(x) \) when \( x = \frac{\pi}{6} \)).
   (b) \( f(x) = x^2 - 4x \) and \( g(x) = 2x \)
   (c) \( f(x) = 7 - x^2 \) and \( g(x) = 2 \)