PROBLEM SETS 3. DUE FRIDAY 8 SEPTEMBER

Problem Set 3. Problems from Lecture 3.

Reading. Quick Calculus, pp. 97–129.

Supplementary reading. Simmons, Chapter 3.

1. Differentiate the following functions, using the rules we learned in lecture today.
   (a) \( y = 3x^4 + 2x^3 - x^2 + 4x - 7 \)
   (b) \( y = (2x^2 + 3)(3x^4 - 2x - 5) \)
   (c) \( y = \frac{3x - 7}{2x^2 + 1} \)
   (d) \( y = (10x - 2)^5(3x^2 - 1)^2 \)
   (e) \( y = \sec \theta \csc \theta \)
   (f) \( y = \tan \theta = \frac{\sin \theta}{\cos \theta} \)
   (g) \( y = e^{5x+7} \)
   (h) \( y = \ln \left( \frac{3x^2}{12x+2} \right) \)
   (i) \( y^3 = \sqrt{2xy - 4xy^2} \)
   (j) \( y = (x^2 + 4)^{\frac{3}{2}} \)

2. Given a cubic equation \( f(x) = ax^3 + bx^2 + cx + d \), for what constants \( a, b, c, \) and \( d \) does the graph of \( f(x) \) have exactly
   (a) two horizontal tangents?
   (b) one horizontal tangent?
   (c) no horizontal tangents?

   (Hint: A horizontal tangent to the graph occurs when the derivative \( f'(x) = 0 \).)

3. Find the values of \( x \) for which the graph \( f(x) = x + 2 \sin(x) \) has a horizontal tangent.

4. Find the tangent line to
   \[ f(x) = \frac{x^3 + x}{x - 1} \]
   at the point \((2, 10)\).

5. We have talked about the tangent line to a graph at some point \( P \) on the graph. The **normal line** to a graph at the point \( P \) is the line that is perpindicular to the tangent line to the graph at \( P \). Given a line \( f(x) = mx + b \), the perpindicular line \( g(x) \) to \( f(x) \) at \( P \) is the line with slope \(-\frac{1}{m}\), also going through \( P \). (See the figure on the next page.)

Find the tangent line and the normal line to the graph \( y = \frac{6}{x+2} \) at the point \((1, 2)\).
Figure 1. This shows the line $f(x) = mx + b$ and the perpendicular line $g(x) = -\frac{1}{m}x + d$, where $d$ is determined by our choice of $P$. 