PROBLEM SETS 1 & 2. DUE THURSDAY 7 SEPTEMBER

Problem Set 1. Problems from Lecture 1.

Reading. *Quick Calculus*, Chapter 1.

Supplementary reading. Simmons, Chapter 1.

1. Given a quadratic equation of the from $ax^2 + bx + c = 0$, we can solve for x using the formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
Using the above formula, solve the equation $4x^2 - 5x - 6 = 0$ for x.
2. Find $f(x)$ if $f(x + 1) = x^2 - 5x + 3$.
3. Graph the following functions, and give their domain and range.
   (a) $y = 3x - 2$.
   (b) $y = 4x^2 + 3$.
4. Graph the following functions, and give their domain and range.
   (a) $y = 2^x$.
   (b) $y = 2^{x+3}$.
5. Graph the following functions, and give their domain and range.
   (a) $y = log_2(x)$.
   (b) $y = log_2(x) + 5$.
6. Graph the following functions, and give their domain and range.
   (a) $y = \sin(x)$.
   (b) $y = \sin(2x)$.
7. Graph the following functions, and give their domain and range.
   (a) $y = \tan(x)$.
   (b) $y = 4\tan(x)$.
8. Simplify the following expressions.
   (a) $\log_{10}(\frac{x+y}{z})$.
   (b) $25^{\log_{25}(x+y)+\log_5(\frac{x}{y})}$.
9. Make the following computations using right triangles.
   (a) For $\theta = \frac{\pi}{4} = 45°$, compute $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.
   (b) For $\theta = \frac{\pi}{6} = 30°$, compute $\sec(\theta)$, $\csc(\theta)$, and $\cot(\theta)$.
10. Simplify the following expressions (i.e. write them in terms of elementary trig functions $\sin(\phi)$, $\sin(\theta)$, etc.).
    (a) $\sin(\theta + \phi)$.
    (b) $\cos(3\theta)$.

Problem Set 2. Problems from Lecture 2.

Reading. *Quick Calculus*, pp. 50–97.

Supplementary reading. Simmons, Chapter 2, sections 2.1–2.5. Read section 2.6 if you are interested in some applications of the derivative.

1. Compute the following limits.
   (a) $\lim_{\theta\to 0} \frac{\sin(5\theta)}{\theta}$
   (b) $\lim_{\theta\to 0} \frac{\sin(3\theta)}{\sin(4\theta)}$
   (c) $\lim_{x\to\infty} \frac{x}{x^2+1}$
   (d) $\lim_{x\to\infty} \frac{2x^2+3x}{3x^2-2}$
2. Where are the following functions discontinuous?
   (a) $\frac{x}{x^2+1}$
   (b) $\frac{1}{x^2+x-6}$
   (c) $\frac{x^3+x}{x^2+1}$
   (d) $\frac{x^2+2x}{x^3+2x^2-x-2}$
3. Use the definition of the derivative to show that for $f(x) = ax^2 + bx + c$, for constants $a, b, c \in \mathbb{R}$, the derivative is $f'(x) = 2ax + b$.
4. Use the definition of the derivative to find the derivative of the function $f(x) = \frac{x}{x+1}$.
5. Use the definition of the derivative and the double angle formula to compute the derivative of $f(\theta) = \cos(\theta)$.
6. Sketch the graph of the following two functions. For each, state where it is **not** differentiable.
   (a) $f(x) = \sqrt{|x|}$.
   (b) $f(x) = |x^2 - 9|$.
7. Let $f(x) = \begin{cases} x^2 & \text{if } x \le -1, \\ mx + b & \text{if } x > -1. \end{cases}$ What must m and b be for $f(x)$ to be differentiable at all points?
8. A penny is dropped off a ledge on the World Trade Center in New York City. The ledge is 1024 feet above the ground. The penny falls a distance of $s = 16t^2$ feet in t seconds.
   (a) How long does the penny fall before it hits the ground?
   (b) What is the average velocity at which the penny falls during the first three seconds?
9. With the same situation as in Problem [**?**], answer the following questions.
   (a) What is the average velocity at which the penny falls during the last four seconds?
   (b) What is the instantaneouse velocity of the penny when it hits the ground?
10. An oil tank is to be drained for cleaning. There are V gallons of oil left in the tank after t minutes of draining, where $V = 50(40 - t)^2$.
    (a) What is the average rate at which oil drains out of the tank during the first 20 minutes?
    (b) What is the rate at which oil is flowing out of the tank 20 minutes after draining begins?