ArsDigita University
Month 2: Discrete Mathematics - Professor Shai Simonson

Syllabus

Week 1: Introduction, Proofs, Logic, Boolean Algebra and applications, Sets and applications, Basic sums and functions.

Reading: Rosen 1.1-1.8, 3.1-3.2, 5.5, 9.1-9.3 How to Read Mathematics (http://academics.stonehill.edu/compsci/History_Math/math-read.htm), Polya, How to Solve It.

Lecture 1: What kinds of problems are solved in discrete math? What are proofs? Examples of proofs by contradiction, and proofs by induction: Triangle numbers, irrational numbers, and prime numbers. (3.1-3.2)

Lecture 2: Boolean Algebra and formal logic. Applications in algorithms, complexity theory, AI, digital logic design and computer architecture. (1.1-1.2, 9.1-9.3)

Lecture 3: More logic: quantifiers and predicates. Sets, operations on sets, using logic to prove identities on sets. (1.3-1.5)

Lecture 4: Sets. Applications in counting (the inclusion-exclusion theorem), theory of computation and data structures. (5.5)

Lecture 5: Growth rate of functions, Big-O notation, Countability, 1-1 correspondence. Applications to algorithms and theory of computation. (1.6-1.8)

Week 2: Induction, recursion, recurrence equations, graphs.

Reading: Rosen 3.3-3.5, 5.1-5.3, 7.1-7.5


Lecture 2: Chinese rings puzzle – Grey codes, graphs, hypercubes, Hamiltonian and Euler circuits, planar graphs, Euler’s theorem. (7.1-7.5)

Lecture 3: Solving recurrence equations – repeated substitution, the Master Theorem with applications to algorithms, change of variable technique. (5.1, 5.3)

Lecture 4: Solving recurrence equations – guessing and proving correct by induction, linear homogeneous types. The Josephus Problem. (5.2)

Lecture 5: Mathematical induction – a flexible and useful tool. Many examples and the idea of strong induction. (3.2)
**Week 3:** Counting and discrete probability. Combinations, permutations, pigeonhole principle, inclusion/exclusion revisited.

**Reading:** Rosen 4.1-4.7, 5.6, How to Read Math (re-read from week 1)

- **Lecture 1:** Combinations and permutations. Pascal’s triangle and binomial coefficients. (4.1, 4.3)
- **Lecture 2:** Counting problems using combinations, distributions and permutations. (4.6)
- **Lecture 3:** The pigeonhole principle and examples. The inclusion/exclusion theorem and advanced examples. A combinatorial card trick. (4.2, 4.7, 5.6)
- **Lecture 4:** Discrete probability, the birthday *paradox*, and many examples. (4.4)
- **Lecture 5:** Conditional probability, and more counting. (4.5)

**Week 4:** Generating functions, Number theory for cryptography and computer science, equivalence relations, partial orders, trees.

**Reading:** Rosen 2.1-2.5, 5.4, 6.1-6.6, 8.1-8.2

- **Lecture 1:** Generating functions. (5.4)
- **Lecture 2:** Primes, Greatest Common Divisors and the Euclidean Algorithm. (2.1-2.5)
- **Lecture 3:** The two-jug puzzle as demonstrated by Bruce Willis in *Die Hard III*.
- **Lecture 4:** Congruences and Fermat’s little theorem. Applications to Cryptography. (6.1-6.6)
- **Lecture 5:** Partial orders, trees and equivalence relations. Applications to algorithms. (8.1-8.2)