ArsDigita University

Month 2: Discrete Mathematics - Professor Shai Simonson

Examination 2 – 100 points

Show all work for partial credit. You may use two hours for this exam. After one hour, raise your hand if you feel that the time constraint will be too tight.

Name: ________________________________________________________________

1. /30
2. /28
3. /28
4. /14

Total: /100
1. Sums (30 points)

a. Calculate a formula for the following sum, and prove your formula in any way you like. \((a + (a+1) + (a+2) + \ldots + (a+b))\).

b. The following sum comes up in the analysis of certain sorting algorithms. Find a closed form expression for \(1*2 + 2*4 + 3*8 + 4*16 + \ldots + n2^n\).
2. (28 points) Short Answers – Countability, Sets, Functions

a. Is the set of all finite binary trees countable? Explain briefly.

b. Prove that \( \log(\log x^k) \) is \( \Theta(\log x) \).

c. Assume a domain and range of the real numbers, and state whether each of the following is a function. If it is, then state whether it is onto and whether it is one-one. If it is both, then write down the inverse function.

i. \( x^3 \)

ii. \( \sqrt{x} \)

ii. The first digit of \( x \) after the decimal point
3. Towers Of Hanoi (28 points)

a. What happens when you try to run the following attempt at a solution to the Towers of Hanoi problem? Explain.

```java
ToH (n, From, To, Using)   {
    if (n>0) {
        ToH(n-1, From, Using, To);
        ToH(1, From, To, Using);
        ToH(n-1, Using, To, From);
    }
}
```

A node in a Towers of Hanoi graph is labeled 23221.

b. How many disks in this problem?

c. Draw a picture of the configuration represented by this node.

d. Is this a node that gets visited during the Hanoi solution from peg 1 to peg 2? Explain your answer.
4. Chinese Rings and Gray Codes (14 points)

   a. What are the two Gray codes that precede and follow 101011?

   b. Assume that 1 means that a ring is around the steel bar, and 0 means that the ring is free. Does 101011 appear in the sequence that solves the six ring Chinese Ring Puzzle? Explain your answer.